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Effects of Thermal Diffusion on Viscoelastic Fluid Flow through a Vertical Flat Plate

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Abstract

Viscoelastic fluid flow through a semi-infinite vertical rigid plate with diffusion-thermo and thermal-diffusion has been studied. To obtain the non-dimensional, coupled non-linear momentum, energy and concentration equations, the usual transformations have been used. The obtained non-dimensional equations have been solved by implicit finite difference technique. The stability and convergence analysis have been analyzed. From the above analysis, parameters restriction have been obtained to calculate the converge results. The effects of the various parameters entering into the problem on the velocity, temperature and concentration are shown graphically. Finally, a qualitative comparison with the published results is shown in tabular form.

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Keywords :Viscoelastic fluid, diffusion-thermo, thermal-diffusion, implicit finite difference method;

1. Introduction

The viscoelastic fluid has been received momentum in the recent past because of its numerous applications in polymer technology, metallurgy, polymer sheet extrusion from a dye, polymer processing industry in particular in manufacturing process of artificial film. The study of boundary layer flow of a viscoelastic fluid through a vertical plate in the presence of Soret and Dufour's effect has wide range of applications in the field of chemical engineering

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and production of synthetic sheets. This is consequential to the production of heavy crude oils by means of thermal process. These oils considered as viscoelastic fluid which has both viscous and elastic property.

Diffusion occurs in a mixture under the presence of temperature gradients even when there are no concentration differences. This process is defined as the thermal-diffusion. In other words it can be said that the thermal-diffusion occurs when mass flux can be generated by a temperature gradient. This effect is also known as Soret effect.

Rajagopal et al. [1] studied the boundary layer flow of a viscoelastic fluid over a stretching sheet. However, Eckert and Drake [2] have showed many cases where Dufour effect cannot be neglected. In this regards, Tsai and Huang [3] investigated the heat and mass transfer for Soret and Dufour's effects on Hiemenz flow through porous medium onto a stretching surface. Recently Damseh and Shannak [4] analyzed that the Viscoelastic fluid flow past an infinite vertical porous plate in the presence of first-order chemical reaction. Sreekanth et al. [5] studied about hydromagnetic natural convection flow of an incompressible viscoelastic fluid between two infinite vertical moving and oscillating plates. Very recent Gbadeyan et al. [6], examined heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid in the presence of magnetic field.

Our aim is to extend the work of Gbadeyan et al. [6] for unsteady case and to solve the problem by implicit finite difference method. In this paper, the work has been done with the effects of both thermal and mass diffusion on two dimensional unsteady flow of an incompressible viscoelastic fluid through a vertical plate.

2. Mathematical formulation

Consider the unsteady two-dimensional laminar flow of an incompressible viscoelastic fluid (obeying second grade model) through a vertical rigid plate with thermal-diffusion and diffusion thermo effects. The positive x coordinate is measured along the plate in the direction of fluid motion and the positive y coordinate is measured normal to the plate. The variable temperature T_w and variable concentration C_w at wall of the plate occupied with viscoelastic fluid of uniform ambient temperature T_∞ and uniform ambient concentration C_∞ also the uniform velocity U_0 . The physical configuration of the above problem is given in Fig. 1.

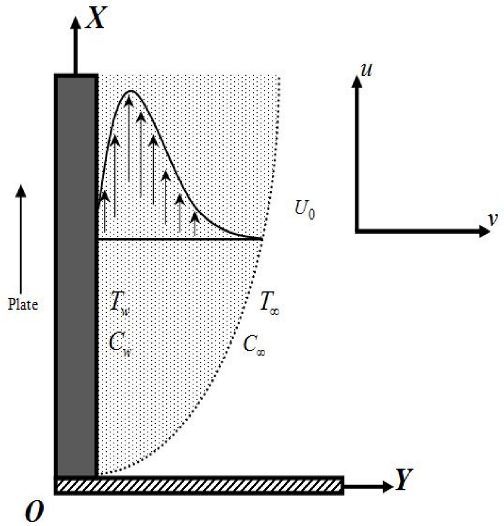


Fig.1. Physical configuration and coordinate system.

The following dimensionless variables that are used to obtained dimensionless governing equations (1)- (4) ;

$$X = \frac{xU_0}{\upsilon}, Y = \frac{yU_0}{\upsilon}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \tau = \frac{tU_0^2}{\upsilon}, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$\phi = \frac{C - C_\infty}{C_w - C_\infty}$$

Using these above dimensionless variables, the following dimensionless equations have been obtained as;

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = G_r\theta + G_m\phi + \frac{\partial^2 U}{\partial Y^2} + K\left[\frac{\partial^3 U}{\partial \tau \partial Y^2} + U\frac{\partial^3 U}{\partial X \partial Y^2} + \frac{\partial U}{\partial X}\frac{\partial^2 U}{\partial Y^2} - \frac{\partial U}{\partial Y}\frac{\partial^2 V}{\partial Y^2} + V\frac{\partial^3 U}{\partial Y^3}\right] \tag{2}$$

$$\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial Y^2} + D_u\frac{\partial^2 \phi}{\partial Y^2} \tag{3}$$

$$\therefore \frac{\partial \phi}{\partial \tau} + U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial Y^2} + S_r\frac{\partial^2 \theta}{\partial Y^2} \tag{4}$$

The corresponding non-dimensional boundary conditions are as;

$$\tau > 0,\ U = 1,\ V = 0,\ \theta = 1,\ \phi = 1, \text{ at } Y = 0 \tag{5}$$
$$U = 0,\ V = 0,\ \theta \to 0,\ \phi \to 0, \text{ as } Y \to \infty$$

The non-dimensional parameters are; Grashof number G_r, Modified Grashof number G_m, Viscoelastic parameter K, Prandtl number P_r, Dufour number D_u, Schmidt number S_c and Soret number S_r.

3. Shear Stress, Nusselt and Sherwood Number

From the velocity, the effects of various parameters on the local and average shear stress have been calculated. The following equations represent the local and average shear stress at the plate. Local shear stress $\tau_L = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$ and average shear stress $\tau_A = \mu\int\left(\frac{\partial u}{\partial y}\right)_{y=0} dx$ which are proportional to $\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$ and $\int_0^{100}\left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX$ respectively. From the temperature field, the effects of various parameters on the local and average heat transfer coefficients have been investigated. The following equations represent the local and average heat transfer rate that is well known Nusselt number. Local Nusselt number, $N_{uL} = \mu\left(-\frac{\partial T}{\partial y}\right)_{y=0}$ and Average Nusselt number, $N_{uL} = \mu\int\left(-\frac{\partial T}{\partial y}\right)_{y=0} dx$ which are proportional to $\left(-\frac{\partial \theta}{\partial Y}\right)_{Y=0}$ and $\int_0^{100}\left(-\frac{\partial \theta}{\partial Y}\right)_{Y=0} dX$ respectively. From the concentration field, the effects of various parameters on the local and average mass transfer coefficients have been analyzed. The following equations represent the local and average mass transfer rate that is well known Sherwood number. Local Sherwood number, $S_{hL} = \mu\left(-\frac{\partial C}{\partial y}\right)_{y=0}$ and Average Sherwood number, $S_{hL} = \mu\int\left(-\frac{\partial C}{\partial y}\right)_{y=0} dx$ which are proportional to $\left(-\frac{\partial \phi}{\partial Y}\right)_{Y=0}$ and $\int_0^{100}\left(-\frac{\partial \phi}{\partial Y}\right)_{Y=0} dX$ respectively.

4. Numerical Analysis

To solve the non-dimension system by implicit finite difference technique, it is required a set of finite difference equations. In this case, the region within the boundary layer is divided by some mash of lines parallel to X and Y axes where $X-$axis is taken along the plate and $Y-$axis is normal to the plate as shown in Fig. 2. Here, the plate of height $X_{\max} = (100)$ i.e. X varies from 0 to 100 and it is assumed that the maximum length of boundary layer is $Y_{\max} = (35)$ as corresponding to $Y \to \infty$ i.e. Y varies from 0 to 35 have been considered. Consider $m = 125$ and $n = 125$ in X and Y directions. It is assumed that ΔX, ΔY are constant mesh sizes along X and Y directions respectively and taken as follows,

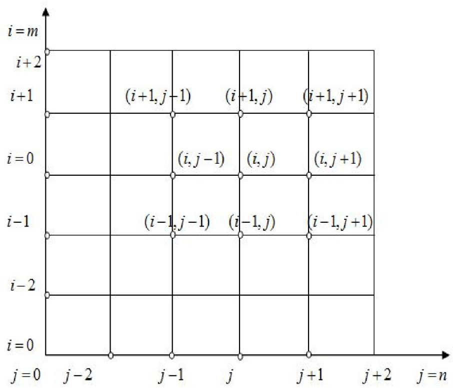


Fig. 2. Implicit finite difference system grid.

$\Delta X = 0.8(0 \le x \le 100)$; $\Delta Y = 0.28(0 \le y \le 35)$ with the smaller time-step, $\Delta\tau = 0.005$.

Let U' , V' , θ' and ϕ' denote the values of U , V , θ and ϕ at the end of a time-step respectively. An appropriate set of finite difference equations have been obtained as;

$$\frac{U'_{i,j} - U'_{i-1,j}}{\Delta X} + \frac{V'_{i,j} - V'_{i,j-1}}{\Delta Y} = 0 \tag{6}$$

$$\frac{U'_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta X} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = G_r\theta'_{i,j} + G_m\phi'_{i,j} + \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2}$$

$$+K\left[\frac{U'_{i,j+1} - 2U'_{i,j} + U'_{i,j-1} - U_{i,j+1} + 2U_{i,j} - U_{i,j-1}}{\Delta\tau(\Delta Y)^2}\right.$$

$$+U_{i,j}\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1} - U_{i-1,j+1} + 2U_{i-1,j} - U_{i-1,j-1}}{\Delta X(\Delta Y)^2} + V_{i,j}\frac{U_{i,j+2} - 3U_{i,j+1} + 3U_{i,j} - U_{i,j-1}}{(\Delta Y)^3}$$

$$\left.+\frac{U_{i,j} - U_{i-1,j}}{\Delta X}\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} - \frac{U_{i,j+1} - U_{i,j}}{\Delta Y}\frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta Y)^2}\right] \tag{7}$$

$$\frac{\theta'_{i,j} - \theta_{i,j}}{\Delta\tau} + U_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta X} + V_{i,j}\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y} = \frac{1}{P_r}\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2} + D_u\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta Y)^2} \tag{8}$$

$$\frac{\phi'_{i,j} - \phi_{i,j}}{\Delta\tau} + U_{i,j}\frac{\phi_{i,j} - \phi_{i-1,j}}{\Delta X} + V_{i,j}\frac{\phi_{i,j+1} - \phi_{i,j}}{\Delta Y} = \frac{1}{S_c}\frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{(\Delta Y)^2} + S_r\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2} \tag{9}$$

with initial and boundary conditions;

$$U^n_{i,0} = 1, V^n_{i,0} = 0, \theta^n_{i,0} = 1, \phi^n_{i,0} = 1 \tag{10}$$

$U^n_{i,L} = 0, V^n_{i,L} = 0, \theta^n_{i,L} = 0, \phi^n_{i,L} = 0$ where $L \to \infty$.

Here the subscripts i and j designate the grid points with X and Y coordinates respectively and the subscript n represents a value of time, $\tau = n\Delta\tau$ where $n = 0,1,2,3,\ldots\ldots\ldots$. The new velocity U' , the new temperature θ' and the new concentration ϕ' at all interior nodal points may be obtained by successive applications of above finite difference equations. The numerical values of the local Shear Stress, Nusselt number and Sherwood number are evaluated by five-point approximate formula for the derivatives and then the average Shear Stress, Current density, Nusselt number and Sherwood number are calculated by the use of the **Simpson's $\frac{1}{3}$** integration formula.

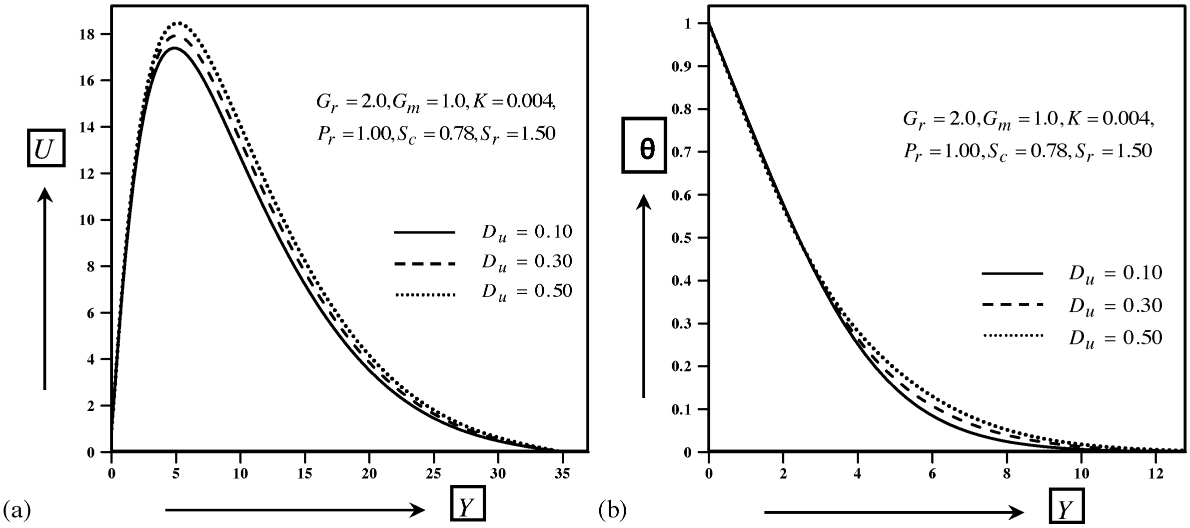
The stability conditions of the problem are as furnished below as;

$$U\frac{\Delta\tau}{\Delta X} + |V|\frac{\Delta\tau}{\Delta Y} + \frac{2}{P_r}\frac{\Delta\tau}{(\Delta Y)^2} \le 1 \text{ and } U\frac{\Delta\tau}{\Delta X} + |V|\frac{\Delta\tau}{\Delta Y} + \frac{2}{S_c}\frac{\Delta\tau}{(\Delta Y)^2} \le 1$$

When $\Delta\tau$ and ΔY approach to zero then the problem will be converged. The convergence criteria of the problem are $P_r \ge 0.13$ and $S_c \ge 0.13$.

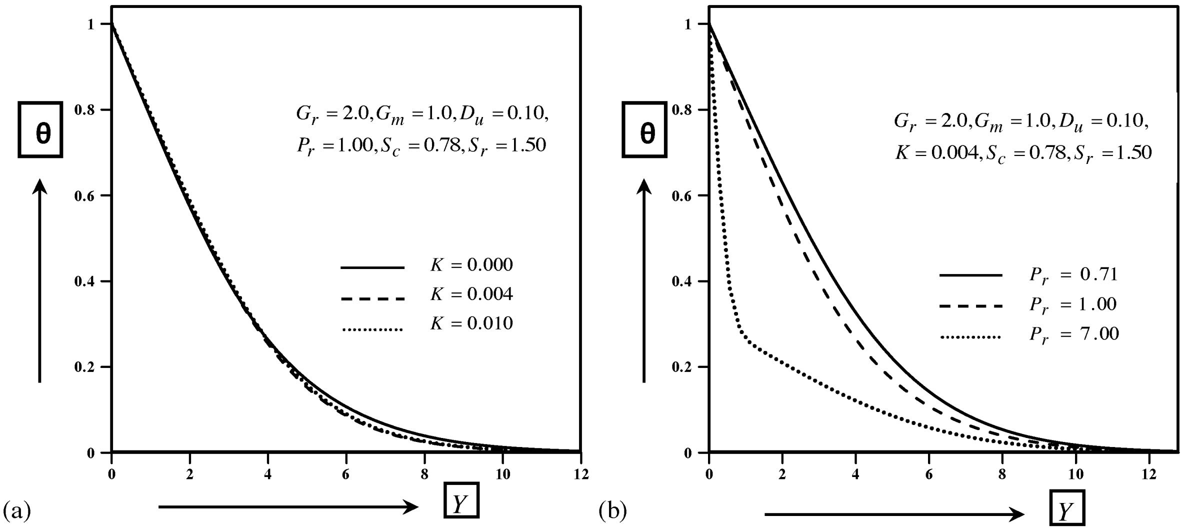
5. RESULTS AND DISCUSSION

To obtain the steady-state solutions, the computations have been carried out up to dimensionless time $\tau = 80$. The results of the computations, however, show little changes in the above mentioned quantities after dimensionless time $\tau = 60$. Thus the solutions for dimensionless time $\tau = 60$ are essentially steady-state solutions. To observe the physical situation of the problem, the steady-state solutions have been illustrated in Figs. 3-7.



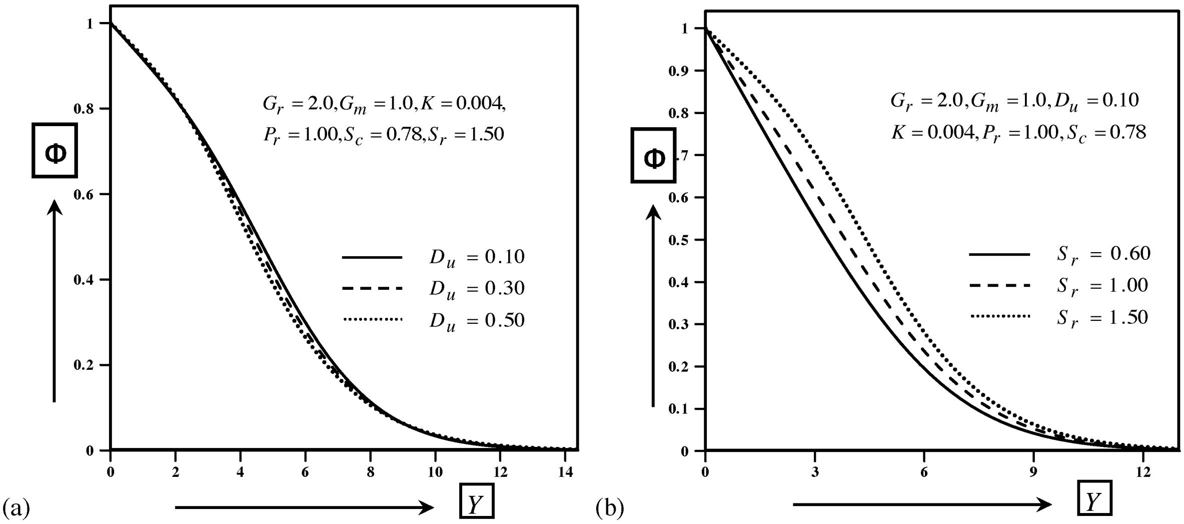
Figs.3 (a) Velocity distributions and (b) Temperature distributions for different values of Dufour number.

The velocity and temperature distributions have been shown in Figs. 3(a) and 3(b) for different values of Dufour number D_u . Both the velocity and temperature distributions increase with the increase of Dufour number D_u . The temperature distributions have been shown in Figs. 4(a) and 4(b) for different values of Viscoelastic parameter K and Prandtl number P_r respectively. In both cases the thermal boundary layers have been decreased with the increase of Viscoelastic parameter K and Prandtl number P_r .



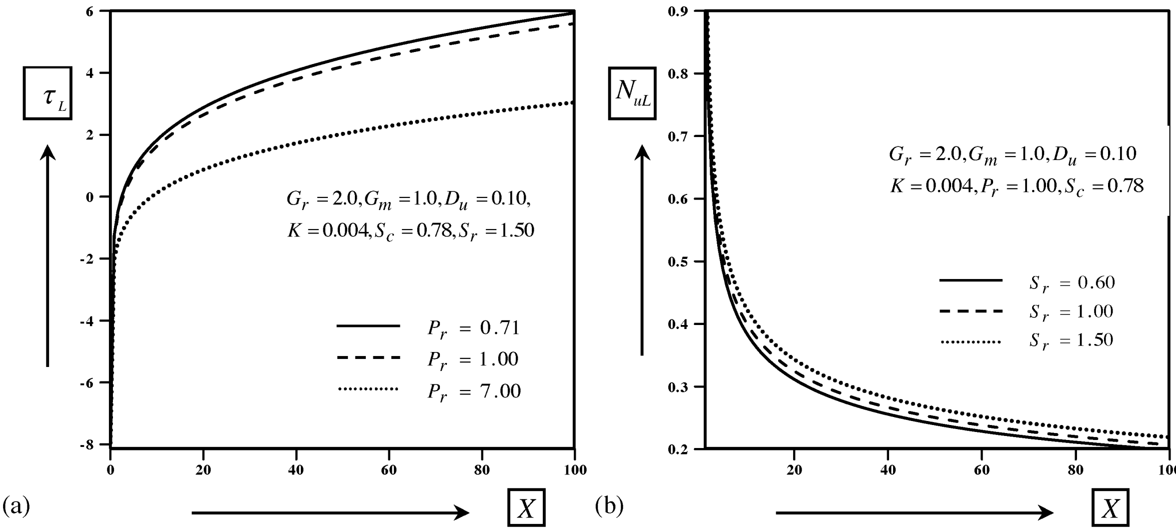
Figs.4 (a) Temperature distributions of for different values of Viscoelastic parameter and (b) Temperature distributions for different values of Prandtl number.

The Concentration distributions have been shown in Figs. 5(a) and 5(b) for different values of Dufour number D_u and Soret number S_r respectively. The concentration distributions decreases with the increase of Dufour number D_u while increases with the increase of Soret number S_r .



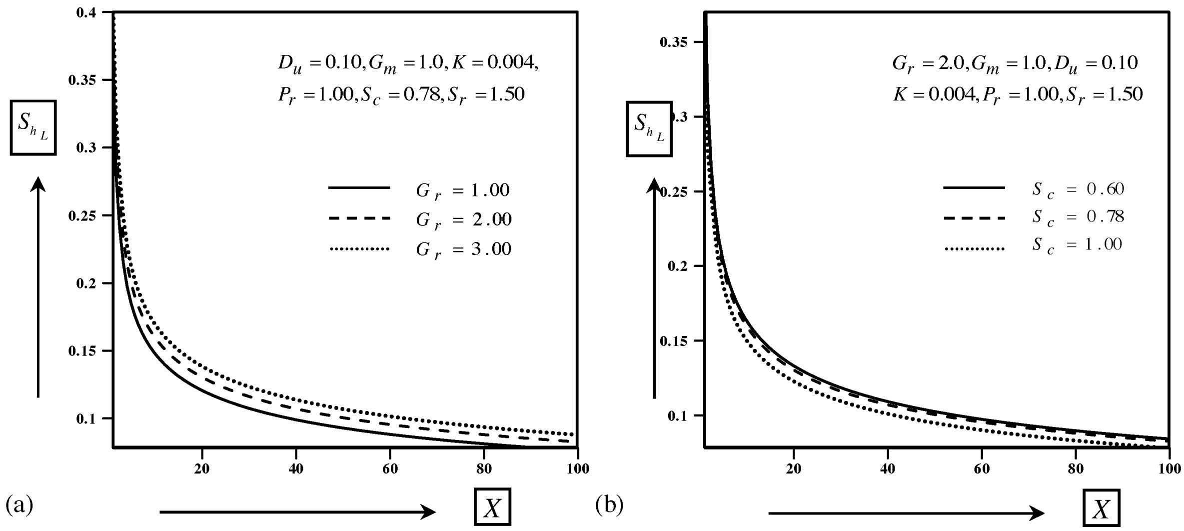
Figs.5 (a) Concentration distributions of for different values of Dufour number and (b) Concentration distributions for different values of Soret number.

The Shear stress for different values of Prandtl number P_r have been plotted graphically in Figs. 6(a). The Shear stress decreases with the rise of Prandtl number P_r. The Nusselt number for different values of Soret number S_r have been plotted in Fig. 6(b) and the Nusselt number increases with the increase of Soret number S_r.



Figs.6 (a) Shear stress for different values of Prandtl number and (b) Nusselt number for different values of Soret number.

The Sherwood number has been shown in Figs. 7(a) and 7(b) for different values of Grashof number G_r and Schmidt number S_c. The Sherwood number increases with the increase of Grashof number G_r while decreases with the increase of Schmidt number S_c.



Figs.7 (a) Sherwood number for different values of Grashof number and (b) Sherwood number for different values of Schmidt number.

Finally, a qualitative comparison of the present steady-state results with the published results (Gbadeyan et al.[6]) is presented in table 1. The accuracy of the present results is qualitatively as well as quantitatively good in case of all the flow parameters.

Table 1.Qualalitive comparison of the present results with the previous results

Increased Parameter	Pervious results given by Gbadeyan et al.[6]			Present results		
	$F'(\eta)$	$\theta(\eta)$	$\phi(\eta)$	U	θ	ϕ
D_u	Inc.	Inc.	Dec	Inc.	Inc.	Dec.
K		Dec.			Dec.	
P_r		Dec.			Dec.	
S_r			Inc.			Inc.

6. Conclusion

In this research work, the implicit finite difference solution of unsteady two-dimensional laminar flow of an incompressible viscoelastic fluid through a vertical plate with Soret and Dufour's effects has been studied. The physical properties are discussed for different values of various parameters and the accuracy of our results is qualitatively good in case of all the flow parameters. Some important findings of this study are given below;

1. For the increase of Dufour number D_u , the velocity, temperature distributions have been increased.
2. Temperature distributions have been decreased with the increase of Viscoelastic parameter K and Prandtl number P_r .
3. Concentration distributions have been decreased with the increase of Dufour number D_u .
4. Concentration distributions have been increased with Soret number S_r .
5. Shear stress has been decreased with the increase of Prandtl number P_r .
6. Nusselt number has been increased with the increase of Soret number S_r .
7. Sherwood number has been increased with the increase of Grashof number G_r while decreased for the increase of Schmidt number S_c .

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